Technical Comments

Comment on "Induced Drag and the Ideal Wake of a Lifting Wing"

W. R. Sears*
University of Arizona, Tucson, Arizona 85721

As stated in Ref. 1, the intent of that exercise was to show how the well-known results concerning induced drag in Prandtl's small-perturbation wing theory, usually derived by energy considerations, could be recovered by a careful force-impulse balance. Since the drag is second order, such first-order effects as the inclination of the plane trailing-vortex sheet have to be taken into account. Since the body of fluid is supposed to be infinitely large, Kelvin's "impulse" concept must be used, because calculations of momentum become ambiguous.

That the static pressure at the Trefftz plane far behind the wing is greater, not smaller, than the atmospheric pressure was an interesting byproduct of the study. It is seen to come from the fact that the wing and its wake, in producing lift, produce not only downward increments of velocity but also small *forward* increments.

McCutchen[2] questions these results and undertakes constructing a different theoretical model. It is difficult to comment on this effort. In the first place, the rolling up of the vortex sheet has nothing to do with this calculation and cannot have any effect on drag to second order; furthermore, some of McCutchen's concepts are foreign to fluid mechanics. He does not make his force-impulse balance clear: the total force on the system should be the net integrated pressure on the boundaries plus the external force being provided at the wing. It should be reflected in a rate of increase of the impulse of the system with consistent signs.

His Eq. (4) is wrong. The flow at the Trefftz plane is two-dimensional and lies in planes perpendicular to the vortex wake, thus inclined at an angle $2\alpha_i$ to the vertical (see Fig. 1, copied from Ref. 1). Thus a, the velocity component in the flight direction, is just $2\alpha_i w$, where w is the vertical component, and its integral is surely not the same as far upstream (viz. zero) if the wing is lifting. The error arises from careless treatment of the approximation of an infinite fluid volume; McCutchen's argument regarding the flux through the system's walls as the volume becomes infinite is a classical blunder. Thus Eq. (6) lacks its leading-order term, as do Eqs. (7) and (8). This is why Eq. (8) states such obvious nonsense—that the rate of change of momentum is the same whether momentum is being added or subtracted. McCutchen explains this by a concept of "forward flow of energy in the wake," which is surely a mystery to this writer—as is the concept of "buckling" of the wake (see Introduction).

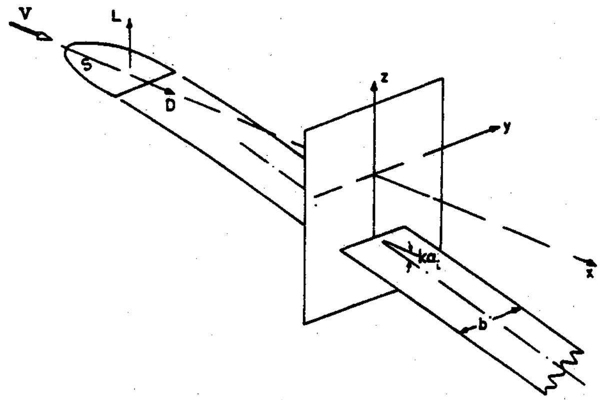

Fig. 1 Wing, trailing-vortex wake, and Trefftz plane.

I believe the little analysis of Ref. 1 is consistent and correct. I also believe that authors should have opportunities to present controversial ideas. But Prandtl's small perturbation wing theory hardly seems like a subject for controversy in 1988.

References

[1]Sears, W. R., "On Calculation of Induced Drag and Conditions Downstream of a Lifting Wing," *Journal of Aircraft*, Vol. 11, March 1974, pp. 191-192.

[2]McCutchen, C. W., "Induced Drag and the Ideal Wake of a Lifting Wing," *Journal of Aircraft*, Vol. 26, No. 5, May 1989, pp. 489-492.

Author's Reply

C. W. McCutchen*
National Institutes of Health, Bethesda, Maryland

PROFESSOR Sears and I have been discussing induced drag for two years. I thank him for his continued interest.

Sears says my Eq. (4) is wrong. He writes, "The flow at the Trefftz plane is two-dimensional and lies in planes perpendicular to the vortex wake, thus inclined at an angle $2\alpha_i$ to the vertical.... Thus a, the velocity component in the flight direction, is just $2\alpha_i w$, where w is the vertical component, and its integral is surely not the same as far upstream (viz. zero) if the wing is lifting." But it is only an assumption that the forward

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*Professor, Department of Aerospace and Mechanical Engineering. Honorary Fellow AIAA.

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*Physicist, National Institute of Diabetes and Digestive and Kidney Diseases. Member AIAA.

component of the flow velocity is $2\alpha_i w$. No physical reason for it was given in Sears' original note.[1] I will show later that the assumption is untenable.

He goes on, "The error arises from careless treatment of the approximation of an infinite fluid volume; McCutchen's argument regarding the flux through the system's walls as the volume becomes infinite is a classical blunder." I presume that Sears suspects a blunder because the limiting value of each integral in Eq. (4) depends on the way in which the outline of the region of integration is expanded to infinity. But because the integrals are over the up and downstream faces of the same cylindrical control volume, the two outlines expand to infinity in the same way. The difference between the integrals, "the flux through the system's walls," is well defined, and becomes smaller and smaller without limit as the control volume is expanded transversely to the flow direction. I showed this in detail in my Note. (I said the derivation in Landau and Lifshitz[2] was faulty because it omits this demonstration.) If there is an error in my reasoning, Sears should point to the exact step where it lies.

I understand Sears' puzzlement over Eq. (8). It is a surprise to find that momentum drag is always negative. But a larger, positive pressure drag always accompanies it.

There is nothing mysterious about forward flow of energy in the wake. The fluid in tip vortex cores is at negative pressure and flows aft.[3] The forward flow of energy by mechanical work is as real as if each vortex were a pipe and had a vacuum pump at its aft end.

Indeed, energy flows forward in Sears's assumed wake, which is at positive pressure and moves forward relative to fluid at infinity. It flows forward both as mechanical work [my Eq. (15)] and by convection of kinetic energy [my Eq. (16)].

Sears finds it a further mystery that a positive integral of pressure across the wake would make it buckle. What else can happen to a limber column put into endwise compression? Can one push an airplane with a towline?

Why does Sears' treatment yield the correct induced drag in the approximation of infinitesimal downwash velocity yet subdivide it wrongly into its pressure and momentum constituents? Sears gives most of the answer himself.[1] His Eq. (7) sets drag equal to the energy of vertical and lateral motion per unit length of wake, and is the classic formula for induced drag in this approximation. It is Landau and Lifshitz's Eq. (47.3),[2] or my Eq. (10) without its $-a^2$ term.

Sears' Eq. (6) is his Eq. (7) with the addition of identical terms on either side of the equals sign. These terms are linear in the longitudinal component of wake velocity. Sears correctly notes that they have no effect on the total induced drag, and only change its apportioning into pressure and momentum constituents.

Sears takes the longitudinal component of wake velocity relative to fluid at infinity to be $k\alpha_i w$. Because the final downwash angle is $2\alpha_i$, he says that $k=2$. This choice makes the component of wake velocity along the axis of the wake's vortex sheet equal to zero. But there is no physical reason for this to be so. Instead, given the assumption that the forward component of wake velocity relative to fluid at infinity is $k\alpha_i w$, continuity requires that $k=0$. My Eqs. (6) and (8) confirm that the longitudinal wake velocity does not appear at first power in either the pressure or the momentum drag. The induced drag is purely pressure drag, in this infinitesimal-loading approximation.

Sears says ". . . Prandtl's small perturbation wing theory hardly seems like a subject for controversy in 1988." I have not found fault with it for calculating lift, and I note that it was Sears, not Prandtl, who used it in calculating induced drag by momentum balance.[1]

At the bottom of the first column and the top of the second column on page 492 in my Note, $2¼$ should have been $2^{1/4}$.

References

[1]Sears, W. R., "On Calculation of Induced Drag and Conditions Downstream of a Lifting Wing," *Journal of Aircraft,* Vol. 8, March 1974, pp. 191–192.

[2]Landau, L. D. and Lifshitz, E. M., *Fluid Dynamics,* Pergamon, London, 1959, pp. 175–176.

[3]Batchelor, G. K., "Axial Flow in Trailing Line Vortices," *Journal of Fluid Mechanics,* Vol. 20, No. 4, 1964, pp. 645–658.